Erratum

IF YOU FIND TYPOS WHICH ARE NOT MENTIONNED HERE PLEASE COMMUNICATE THEM TO ME!!

These typos were mostly found by Percy G. Li during his careful reading of the book, and I like to express my gratitude here.
Please do not be concerned that some entries are distinguished by a *.

Page 14, Exercise 1.4.1. I am overly optimistic here. I am assuming that the reader knows the following fact from integration theory: If a (reasonable) function f defined on an interval I is such the $\int_I \mathrm{d}x f(x)\xi(x) = 0$ for each function ξ which is continuous and with support in I then f is zero a.e. on I.

Page 29, equation (2.9). I should have stressed the simple fact that if the operator A corresponds to the observable $\mathcal{O}$ then the operator A^2 corresponds to the observable $\mathcal{O}^2$ (with the obvious definition of $\mathcal{O}^2$).

Page 42, line 5 please read ...which is not the case of the "function δ_x".

Page 63, four lines above (2.86) please read ' "multiplication by $\exp(\mathrm{i}tp^2/(2m\hbar))$" '.

Page 85, line -6: Please read $U(t) = \exp(-\mathrm{i}tH)$.

Page 115, Exercise 4.4.4. The definition of $\mathcal{H}$ should be the definition of $\mathcal{H}'$ and conversely.

Page 122, top. More explicitely, on the one hand, the unitary transformations corresponding to time translations represent the time-evolution of the system. On the other hand, Lorentz transformations can be thought of as changes of coordinates, which are reflected in the state space by the corresponding unitary transformations.

Page 128. The title of Section 4.10 should be: **The states $|\boldsymbol{p}\rangle$ and $|p\rangle$**.

Page 135, equation (5.9). If one requires that $\varphi(f)$ is self-adjoint for $f \in \mathcal{S}$ one must have $\lambda = \bar{\tau}$. This remark is however useless because the property that $\varphi(f)$ is self-adjoint for $f \in \mathcal{S}$ is accidental, and many fundamental fields do not satisfy it.

Page 136, footnote 11. The first claim is nonsense, the field (5.9) always satisfies (5.10) (by the same argument as in the proof of Theorem 5.1.5). However, in the case where $\varphi(f)$ need not be self-adjoint for $f \in \mathcal{S}$, the identity (5.10) is not the proper definition of microcausality, see equation (10.39), and it is true that the field (5.9) satisfies (10.39) only when $|\lambda| = |\tau|$.

Pages 146 and 147. In equations (6.1) and (6.4) replace $=$ by $:=$.

Page 163, equation (6.57). This equation is correct, but it is needed only in the case $v = u$. Besides, there is bungled sentence. Instead of "Using integration by parts..." just above (6.57) please read:

The function g_k is an eigenvalue of the Laplacian, of eigenvalue $-\boldsymbol{k}^2/\hbar^2$, as expressed in (6.7). Using integration by parts

$$\int_B \mathrm{d}^3\boldsymbol{x} \sum_{1\le\nu\le3} (\partial_\nu u(\boldsymbol{x}))^2 = -\int_B \mathrm{d}^3 u(\boldsymbol{x}) \sum_{1\le\nu\le3} \frac{\partial^2 u}{(\partial x_\nu)^2}(\boldsymbol{x}),$$

and that the functions g_k form an orthonormal basis, it is straightforward to formally express the Hamiltonian as...

Page 186, proof of Lemma 8.1.9 and Exercise 8.1.10. It would be more in line with the mainstream terminology to call a Hermitian operator whose eigenvalues are ≥ 0 a positive *semi-definite* Hermitian operator rather than a positive Hermitian operator. (On the other hand, as defined on page 194, a positive *definite* Hermitian operator has all its eigenvalues > 0.)

page 201 I am grateful to Jinhyun Jung for having pointed out to me that physicists do not define the angular momentum by (8.35) but by the opposite operator, so that (8.35) has to be replaced by

$$J_3(x) = -\lim_{\theta\to 0} \frac{\hbar}{\mathrm{i}\theta}(V(\theta)(x) - x) \; . \qquad (*)$$

The reason for this convention is have the formula $V(\theta) = \exp(-\mathrm{i}\theta J_3/\hbar)$ which parallels the formulas (2.72) and (2.75). Note that despite the change of sign between (8.35) and (*) the result of Exercise 8.8.2 is correct (there was a mistake in the original formulation). In order to bring in line with (*) a number of related definitions (in Exercise 9.7.3) a number of sign changes are required in Sections 9.7 and 9.8. I have listed them below, and I have also provided a revised version of these sections.

(Unfortunately, the page numbers are those of my version and do not coincide with those of the printed version.)
Page 201, third line of Exercise 8.8.2, replace $j/2-k$ by $\hbar(j/2-k)$.

Page 220, equation (9.33). Please read $C \in SL(2,\mathbb{C})$ rather than $C \in SU(2)$. The implication is true for all $C \in SL(2,\mathbb{C})$ and later in the proof it is used that way.

Page 239, four lines from bottom. Instead of
"It holds that $\gamma_\mu S(A) = S(A)\kappa(A)^\nu{}_\mu\gamma_\nu = S(A)\gamma_\nu\kappa(A)^\nu{}_\mu$"
please read
"It holds that $\gamma_\mu S(A) = S(A)\kappa(A^{-1})^\nu{}_\mu\gamma_\nu = S(A)\gamma_\nu\kappa(A^{-1})^\nu{}_\mu$".

Page 224, Definition 9.6.2. Replace the definition by $\hat{\pi}_j(A) = a^j$ and in the line below remove "The reason for the minus sign will appear later".

Page 225, in (9.42) change the exponent j into $-j$.

Page 226. In the proof of Proposition 9.6.6, change the sign of all the exponents containing a j.

page 226. In the defintion of $\mathcal{H}_j$ replace $f(\theta v) = \theta^j f(v)$ by $f(\theta v) = \theta^{-j} f(v)$.

Page 227, line 8, replace $\pi_{0,-j}$ by $\pi_{0,j}$.

Page 227, line 15, at the end of the line write now $V(A)(g) = a^j g = \hat{\pi}_j(A)g$.

Page 227, line 17, replace $\hat{\pi}_{-j}$ by $\hat{\pi}_j$. Line 18, replace $\pi_{0,-j}$ by $\pi_{0,j}$.

Page 227, last line, replace $\pi_{0,-j}$ by $\pi_{0,j}$.

Page 228, first line, replace $\pi_{0,j}$ by $\pi_{0,-j}$.

Page 228, last line of Proposition 9.6.13, replace $\pi_{0,-1}$ by $\pi_{0,1}$.

Page 229, line below (9.55). In the sentence commencing there, say: "The case of $\pi_{m,j}$ corresponds to the choice $V = \pi_j$. Then the overwhlming ...."

Page 230, line 2, replace $\exp(\mathrm{i}j\theta/2)$ by $\exp(-\mathrm{i}j\theta?2)$.

Page 230, line 15. Replace the expression $U(0,A)(\varphi)(p) = \exp(\mathrm{i}j\theta/2)$ by $U(0,A)(\varphi)(p) =$

$\exp(-\mathrm{i}j\theta/2)$.

Page 230, in each of the last two lines before the statement of Exercise 9.7.3 a minus sign has to be added in an exponent.

Page 230 a minus sign has to inserted in the definition (9.59).

Page 231, a minus sign has to be inserted in the definition (9.60).

Page 231, line 5, replace $\pi_{0,1}$ by $\pi_{0,-1}$.

Page 240, Exercise 9.11.3, second line, read "so that $\mathrm{i}\hbar\widehat{\partial_\mu(f)}(p) = p_\mu \hat{f}(p)$."

Page 301, equation (11.8) please read

$$(H_0 - \lambda_0 \mathbf{1})w_1 = -(H_I - a_1\mathbf{1})v_0 = \cdots$$

and the line below please read:
$\cdots$ on which $H_0 - \lambda_0\mathbf{1}$ is $\cdots$

Page 305, in equation (11.27) the last integral should be $\int_0^{\theta_1} \mathrm{d}\theta_2 \tilde{H}_1(\theta_2)$.

Page 309, the sentence above (11.44) can be confusing. It simply tries to explain what happens in (11.43). Comparing with (1.13), (11.44) is simply a heuristic reformulation of (11.44).

Page 326, second line of Exercise 12.2.3. In momentum space, H_0 is multiplication by $\boldsymbol{p}^2/(2m)$ so that $U_0(T) = \exp(-\mathrm{i}tH_0)$ is multiplication by $\exp(-\mathrm{i}t\boldsymbol{p}^2/(2m))$ (and not as stated by $\exp(\mathrm{i}t\boldsymbol{p}^2/(2m))$)

Page 337, Theorem 12.4.1. Two lines after (12.32) please read
"... that the function θ occurring there is zero in a neighborhood of the z axis."

Page 337, footnote 34 is nonsensical. Please ignore it.

Page 338, 339. Several occurrences of the expression "the right hand side of (12.39)" have to be replaced by the expression "the quantity (12.40)".

Page 344, It is nonsense to say that $a_i^\dagger(\boldsymbol{p})|0\rangle$ is just a notation, see (12.58) on page 345. Starting with the third sentence of the second paragraph, replace the text with the following: A natural continuous basis describing one-particle states consists of the elements $a_i^\dagger(\boldsymbol{p})|0\rangle$. Here i accounts for the particle type (for simplicity we will consider only spinless particles, otherwise we should also account for the spin of the particle) and for $\boldsymbol{p} \in \mathbb{R}^3$ the operator $a_i^\dagger(\boldsymbol{p})$ is defined as in (5.28). The quantity $a_i^\dagger(\boldsymbol{p})|0\rangle$ is an improper state which makes sense only when integrated against a test function. To describe incoming two-particle states we will similarly use the continuous basis $a_{i_1}^\dagger(\boldsymbol{p}_1)a_{i_2}^\dagger(\boldsymbol{p}_2)|0\rangle$ which represents....

Page 344, line above (12.46) please replace $\varphi = |a_1^\dagger(\boldsymbol{p}_1)a_2^\dagger(\boldsymbol{p}_2)|0\rangle$ by $\varphi = a_1^\dagger(\boldsymbol{p}_1)a_2^\dagger(\boldsymbol{p}_2)|0\rangle$.

Page 345, (12.58) is not an assumption, it is written on top of page 141.

Page 353, line 5. Please read:
and the Hamiltonian H_I is the operator V "multiplication by the function $V(\boldsymbol{x})$"...

Page 391, line 12. Please read: "We assign a factor $-\mathrm{i}g$.." (rather than $\mathrm{i}g$)

Page 391, line -4 and -8, please read "$\delta^{(4)}(\Sigma_v)$" rather than $\delta(\Sigma_v)$.

Page 392, please replace the second and third lines of 13.17 by the following:
...to scattering amplitudes that involve *divergent diagrams*, i.e. diagrams whose value is given by a divergent integral.[1] In this section we start the task of renormalizing the scattering amplitude (13.96) in φ^4 theory, computed at order 2 in perturbation theory.

Page 407, Figure 13.15. The first diagram on the left is obviously wrong, since each diagram should have two incoming external edges and two outgoing external edges. To get the correct diagrams (there are two of them) rotate the figure 90 degrees, and pick two of the four external edges as incoming edges and two as outgoing edges. There are two cases, depending on whether the lonely external edge is incoming or outgoing.

Page 407, second line from bottom, please read: ...obvious now but will appear...

Page 428, line 3, please read "left hand side of (14.7) when $|\xi\rangle = |\psi\rangle = 0$ is about..."

Page 449, line 10. "According to (4.70)..." This is perfectly correct, this refers to

[1]Please note the in the definition of a divergent diagram we need not distinguish between contraction and Feynman diagrams as the values of these differ just by a combinatorial factor. Note also that the objective is not necessarily to assign a value to each diagram, but only to certain sums of these diagrams.

(4.70) page 128, but one has a tendency to read (14.70) instead of (4.70), and this is terribly confusing since (14.70) has nothing to do here.

Page 455, line above (14.90) please read $\langle\Omega|\mathcal{T}\varphi(y_1)\varphi(y_2)|\Omega\rangle$.

Page 463, equation (14.112). Of course here $p_j^2 = \mu^2$ and the limit is taken for $q_j^2 \neq \mu^2$ and $q_j^2 \to \mu^2$. In the right-hand side of the equation, the quantity $a^\dagger(p_1)a^\dagger(p_2)|0\rangle$ stands for an in-state "which before the interaction looks like it consists of a particle of four-momentum p_1 and a particle of four-momentum p_2."

Let us face it: this is not an easy result. The main difficulty is that it no longer works to be cavalier about the in and the out states and pretend as we did on page **??** that $\mathcal{H}_{\mathrm{part}} \subset \mathcal{H}$. One has to find a way...

Page 478, line before Lemma 15.1.13, please read "is spanned by a subset of the canonical basis".

Page 484. The quantity $\Sigma = \Sigma_v$ involved in the Feynman rules and defined on the bottom of page 390 is the sum of the momenta *leaving* the corresponding vertex v, since the momentum corresponding to a line oriented towards that vertex is given a minus sign, while the momentum of a line oriented away for that vertex is given a plus sign. So with the definition (15.30), the quantity Σ_v equals $-\mathcal{L}(x)_v + w_v$. This inconsistency has no serious consequence (besides being confusing). Indeed, the δ function satisfies $\delta^{(4)}(\Sigma) = \delta^{(4)}(-\Sigma)$ so that the formula (15.31) is correct.

*Page 484 line -2, please read (15.32) instead of (15.32).

Page 489, paragraph before equation (15.42). To make matters clearer, to each vertex of the graph we attach a new edge (the "feeding edge"), connected at the other end to a source/sink of electrical current. The electrical network consists of the graph together with the feeding edges, through which electrical current can leave or enter the network.

Page 489, equation (15.42). Defining $\mathcal{L}_0(x)_v$ as the amount of current *leaving* the network through the feeding edge connected to the vertex v is consistent with the definition on top of page 484 with considers the amount of current *entering* the vertex v through the adjacent edges, since these two amounts are equal by conservation of current.

*Page 498, last equation, the middle factor should be $f(-q + (p_1 + p_2)/2)$. (Note however that this is just a formal change since $f(\ell) = f(-\ell)$.)

*Page 498, Figure 6.1, diagram (a), the flow on the edge from vertex 3 to vertex 2 should be $q - (p_2 - p - 1)/2$. There are many other possible for the flows on the edges. It would have been clearer to use the use the canonical flows given later, where the flow on the edge from vertex 1 to vertex 2 is $q + (2p_1 + p_2)/3$, on the edge from vertex 1 to vertex 3 is $-q + (2p_2 + p_3)/3$ and on the edge from vertex 3 to vertex 2 is $q - (p_2 - p1)/3$. But all the possible choices give the same integral in the last line of page 498, as is seen by a translation in the space $\mathbb{R}^{1,3}$.

*Page 499, formula (16.5): to understand the strange notation, think $q = k_1, \ell = k_2$.

*Page 501, top. Going back to viewing a graph as an electrical network as on page 489, when the edges carry numbers (instead of four-momentum) canonical flows have a clean physical interpretation. Assuming as natural that each edge has the same resistance, the possible flows of current through the edges are exactly the canonical flows. This is related to the following principle. Given the amount of current which leaves or enters the network at each vertex, the actual flow of current $(x_e)_{e\in\mathcal{E}}$ minimizes the power spent in the network, power which is proportional to $\sum_{e\in\mathcal{E}} x_e^2$.

*Page 509, bottom, discussion of diagram (a) of Figure 16.1. According to the definitions, one should use the canonical flows, not those written on the figure, for which the values are $k_{1,2} = q+(2p_1+p_2)/3$, $k_{2,3} = -q+(p_2-p_1)/3$, $k_{3,1} = -q-(p_1+2p_2)/3$. This gives the same value for $\bar{k} = (k_{1,2} + k_{2,3} + k_{3,1}$ (Please convince yourself that this has to be the case!)

Page 740, first column, line -11, please read “momentum state space.”

The next page numbers refer to the solution of the exercises, which is online on CUPs’ web site.

Page 825, there are mistakes in the solution of Exercise 6.9.5. (I am gratefull to Jinhyun Jung for pointing this out.) Using the first displayed equation on page 825 and applying formula (4.36) we obtain

$$A\partial B/\partial t = \frac{c\mathrm{i}}{\hbar}\iint \frac{\mathrm{d}^3\boldsymbol{p}}{(2\pi\hbar)^3 2\omega_{\boldsymbol{p}}}\frac{\mathrm{d}^3\boldsymbol{p}'}{(2\pi\hbar)^3 2\omega_{\boldsymbol{p}'}} p'^0 \exp(\mathrm{i}(x, p+p')/\hbar) f^+(p) g^+(p') ,$$

where $p = (\omega_{\boldsymbol{p}}, \boldsymbol{p})$ (see section 4.4) so that $p^0 = \omega_{\boldsymbol{p}}$, and similarly for p'. Thus

$$A\partial B/\partial t = \frac{c\mathrm{i}}{2\hbar}\iint \frac{\mathrm{d}^3\boldsymbol{p}}{(2\pi\hbar)^3 2\omega_{\boldsymbol{p}}}\frac{\mathrm{d}^3\boldsymbol{p}'}{(2\pi\hbar)^3} \exp(\mathrm{i}(x, p+p')/\hbar) f^+(p) g^+(p') .$$

Integrating in $\mathrm{d}^3\boldsymbol{x}$ and using the formula $\int \mathrm{d}^3\boldsymbol{x}\exp(\mathrm{i}(\boldsymbol{x},\boldsymbol{p}+\boldsymbol{p}')/\hbar) = (2\pi\hbar)^3\delta^{(3)}(\boldsymbol{p}+\boldsymbol{p}')$ we obtain

$$\int \mathrm{d}^3\boldsymbol{x} A\partial B/\partial t = \frac{c\mathrm{i}}{2\hbar}\iint \frac{\mathrm{d}^3\boldsymbol{p}}{(2\pi\hbar)^3 2\omega_{\boldsymbol{p}}}\mathrm{d}^3\boldsymbol{p}'\delta^{(3)}(\boldsymbol{p}+\boldsymbol{p}')\exp(\mathrm{i}x(p^0+p'^0)/\hbar)f^+(p)g^+(p').$$

For $\boldsymbol{p}' = -\boldsymbol{p}$ we have $p'^0 = p^0$ and $p' = \bar{p} := (\omega_{\boldsymbol{p}}, -\boldsymbol{p}) = (p^0, -\boldsymbol{p})$ so that

$$\begin{aligned} \int \mathrm{d}^3\boldsymbol{x} A\partial B/\partial t &= \frac{c\mathrm{i}}{2\hbar}\int \frac{\mathrm{d}^3\boldsymbol{p}}{(2\pi\hbar)^3 2\omega_{\boldsymbol{p}}}\exp(2\mathrm{i}x_0p^0/\hbar)f^+(p)g^+(\bar{p}) \\ &= \frac{c\mathrm{i}}{2\hbar}\int \mathrm{d}\lambda(p)\exp(2\mathrm{i}x_0p^0/\hbar)f^+(p)g^+(\bar{p}) \ . \end{aligned}$$

Exchanging A and B and using the transformation $p \to \bar{p}$ which preserves λ_m we find that this equals $\int \mathrm{d}^3\boldsymbol{x}\partial A/\partial t B$. Proceeding in a similar fashion for the other terms one gets

$$\int \mathrm{d}^3\boldsymbol{x} A \overset{\leftrightarrow}{\partial_t} B = \frac{c\mathrm{i}}{\hbar}\int \mathrm{d}\lambda_m(p)(f^-(p)g^+(p) - f^+(p)g^-(p)) \ .$$

Page 827, solution of Exercise 8.5.3. It is challenging to produce a beautiful geometric picture, but I realized after finishing the book that there is a way to look at this which makes the result trivial to understand (but not necessarily to visualize). The basic observation is that if for a unit vector $\boldsymbol{v}$ we denote by $R_{\boldsymbol{v},\theta}$ the rotation of angle θ around the axis determined by $\boldsymbol{v}$, then for any unit vectors $\boldsymbol{u},\boldsymbol{v}$, the loop $R_{\boldsymbol{v},4\pi\theta}, 1/2 \le \theta \le 1$ can be continuously deformed into the loop $R_{\boldsymbol{u},4\pi\theta}, 1/2 \le \theta \le 1$. This is done simply by moving continuously the axis of rotation from $\boldsymbol{u}$ to $\boldsymbol{v}$. As a special case, the loop $R_{4\pi\theta}, 1/2 \le \theta \le 1$ can be continuously deformed into the loop $R'_{4\pi\theta}, 1/2 \le \theta \le 1$, where R'_θ now denote the rotation of angle θ around the third axis oriented *upside down*, which is the same as the rotation of angle $-\theta$ around the third axis, and also the same as the rotation of angle $4\pi - \theta$ around this third axis. Consequently the loop $R_{4\pi\theta}, 0 \le \theta \le 1$ can be deformed continuously in the loop $S_{4\pi\theta}, 0 \le \theta \le 1$ where S_θ is the rotation of angle θ around the third axis if $0 \le \theta \le 2\pi$ and is the rotation of angle $4\pi - \theta$ if $2\pi \le \theta \le 4\pi$ around the same axis. But it should then be obvious how to contract the loop $S_{4\pi\theta}, 0 \le \theta \le 1$.

Page 828 solution of Exercise 8.8.1. The reference given (that the computation is done on page 713) is nonsensical. The computation goes as follows. According to (8.23) for $A = \exp(-\mathrm{i}\theta\sigma_3/2)$ then $\kappa(A)$ is the rotation R_θ of angle θ around the z axis, so that (8.33) shows that

$$V(\theta)(\varphi)(\boldsymbol{p}) = \varphi(R_\theta^{-1}(\boldsymbol{p})) = \varphi(p^1\cos\theta + p^2\sin\theta, -p^1\sin\theta + p^2\cos\theta, p^3)$$

and using (*), the corrected version of (8.35) this yields the formula $J_3(\varphi)(\boldsymbol{p}) = -\mathrm{i}\hbar(p^2\partial_1\varphi(\boldsymbol{p}) - p^1\partial_2\varphi(\boldsymbol{p}))$.

page 829 line 6. Two occurrences of $\kappa(A)$ should be $\kappa(A^{-1})$. Furthermore, the statement of Exercise 8.8.2 is wrong, the wave function is multiplied by a phase $\exp(\mathrm{i}\theta(j/2-k))$ (as the proof shows), not $\exp(-\mathrm{i}\theta(j/2-k))$. However, the statement of Exercise 8.8.2 becomes correct when we amend (8.35) by adding a minus sign as explained above.